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Semiotic Alternations with the Yupana Inca Tawa Pukllay in the Gamified Learning of Numbers at a Rural Peruvian School

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ABSTRACT: Yupana Inca Tawa Pukllay (YITP) is a ludic didactic resource based on semiotic alternation that, using the reading of numbers in the Inca numeral system, improves its equivalent Indo-Arabic reading. Twelve children from first to fourth grade of a bilingual (Spanish-Quechua), multi-grade elementary school in a small rural Peruvian community were assigned an electronic tablet with YITP and learned autonomously, without teachers during the COVID-19 pandemic. The results obtained show that: (a) they learned in a very short period of time (14 min - 05h 41 min) (b) they improved digit reading accuracy on the first attempt (c) they improved digit reading speed d) they achieved a high percentage of correct reading of numbers containing at least one zero digit. The results suggest the potential of YITP as an educational tool in the teaching-learning process of arithmetic.

Keywords: Semiotic alternations, Yupana Inca Tawa Pukllay, Ethnomathematics gamification, Numeral system, Zero

1. Introduction

Rural education is offered in rural territories and cultures located in different regions of the country that present environmental, linguistic, geographical, historical, and cultural particularities. Schools usually serve very small groups of students (Figueroa et al., 2021) with associated problems such as poverty, isolation, accessibility, available resources, etc.

The Peruvian Ministry of Education (MINEDU, 2021) has established a competency-based model for the area of mathematics, with quantity problem solving as the first competency. However, it maintains the traditional dynamics of classes focused mainly on grades rather than on the teaching-learning process. The didactic materials are maintained without the support of appropriate technology according to the new pedagogical strategies and objectives, and traditional teaching dynamics are maintained. In the results obtained from the PISA tests from 2015 to 2018, a slight improvement is observed but the most basic levels of performance in mathematics are maintained (MINEDU, 2018).

The health emergency caused by the COVID-19 pandemic led the temporary closure of schools in both urban and rural areas, forcing students to interrupt their on-site studies and opt for the emergency solution implemented by the Peruvian government: “Aprendo en casa” (“I Learn at Home”) program, which was launched even without the minimum conditions for adequate distance education, seeking to adapt materials, content and media to the national curriculum for basic education (Andrade & Guerrero, 2021) but without an adequate strategy in the use of educational technology. During the last two years, mainly rural students have not had access to classes in multigrade schools, have lacked the presence of teachers and/or adequate feedback on their progress during the deployment of such government programs.

Migrating from a face-to-face educational system to a distance learning system requires not only the acquisition of materials and media, but mainly the use of innovative and adequate methods and methodologies for this purpose, which if necessary use semiotic alternations that allow a more effective learning process and if possible, accelerate it through the development of educational materials that involve the student, capturing their attention by integrating the cultural, scientific and technological elements related to their genuine interest.

The learning of the numeral system, in the case of the first grades of primary education, is an essential basis for students’ understanding of arithmetic and consequently of mathematical skills in general, as well as for their applications throughout their daily activities. This context raised the following research questions for us:

Given the negative circumstances: (a) 1st and 2nd year children never received previous face-to-face education at school, (b) YITP is a new tool inserted in primary education, (c) students have no digital experience, (d) most of students' parents are illiterate, (e) inexistence of wifi connection in the community and isolation due to the pandemic made difficult hardware and software support; will children of a multi-grade school in the Cañaris community of Peru learn to read Indo-Arabic numbers and their equivalent Inca numbers through the semiotic alternation of Yupana Inca Tawa Pukllay (YITP) embedded in an electronic tablet ?

In the learning process of Indo-Arabic and Inca number systems reading using the digital YITP serious game in tablets, what peculiarities will be shown by:

- 1st and 2nd grade children who never had previous face-to-face arithmetic education and
- 3rd and 4th grade children who did have previous face-to-face arithmetic education at school?

What differences in the learning process of Indo-Arabic and Inca number systems reading using the digital YITP serious game in tablets between 1st and 2nd grade versus 3rd and 4th grade will there be?

The present research was developed in a context of geographic isolation, social interactions, and socioeconomic isolation of the Cañaris community, accentuated by the confinement of epidemiological policies in the face of the COVID-19 pandemic, with the absence of a multigrade teacher to teach the reading of Indo-Arabic numbers in arithmetic for elementary school children. It was a quantitative correlational type research with an epistemological, semiotic and ethnomathematical approach to the learning process of children in the numeral system through the use of the Yupana (Inca abacus) and its arithmetic method called YITP, for which the objectives were: (1) Demonstrate that YITP within the Tablet (SERO-TP) will facilitate learning Indo-Arabic numeracy in children with relative isolation due to the COVID-19 pandemic. (2) To compare different parameters of the learning of one-digit to five-digit numbers reading in two groups of children, group a: 1st and 2nd grade (never assisted classes at school before) and group b: 3rd and 4th grade (who assisted classes at school for 1 or 2 years before pandemics), using the SER0-TP. (3) To provide empirical evidence that the didactic use of semiotic alternations as a resource for mathematical learning and self-learning, in this case, the learning of numbers in the Indo-Arabic system by the non-Indo-Arabic YITP, is an intuitive educational support, playful, which facilitates learning and shortens the time in which arithmetic contents are learned and mastered. (4) To provide empirical evidence that the yupana inca, developed centuries ago in Peru and recently proposed as the YITP method is a didactic resource easy to use by children, and therefore, useful, economical, and easy to implement for the current teaching of arithmetic in primary school, with relative independence of the socioeconomic, linguistic, and cultural condition of the students. (5) To provide empirical evidence for our proposal that the YITP has the symbolic representation of place-valued zero in a visuospatial matrix, and that its visuospatial representation, used as a semiotic alternation, facilitates the mastery of learning Indo-Arabic numbers with place-valued zeros.

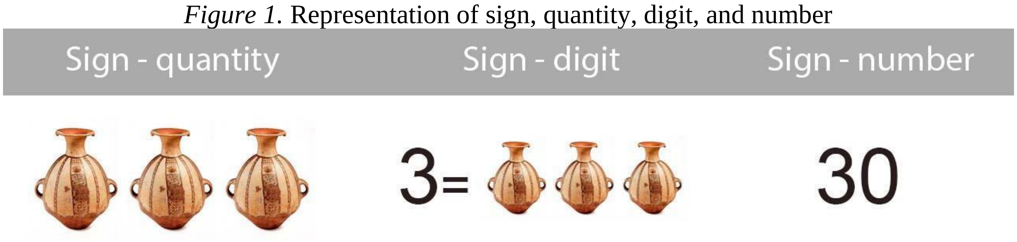
2. Literature review

Semiotic alternations (SA) are the change of one sign for another to represent the same meaning. Their main effect is to make it more precise, or to expand it, or to clarify it (Escotto-Córdova, 2021). SAs enhance thinking, facilitate the generalization of concepts and are a didactic resource used daily in the teaching and learning of any subject of knowledge, and mathematics is no exception. Examples of semiotic alternations are metaphors, drawings, objects, body movements accompanying speech, diagrams, writing, still or moving images in videos, etc.

The key to understanding the use of semiotic alternations as a didactic resource is to be precise in the concepts of sign and meaning that we use. By sign we will understand any physical entity that stands in place of something (a physical entity or a conceptual entity) for someone. The physical form of signs is varied: phonic, gestural, facial, manual or corporal; objectual, wavelengths, non-iconic graphics (e.g., writing), iconic graphics (e.g., drawings). We will understand by meaning everything that is substituted by a sign, be it a physical or conceptual entity. There is no sign without meaning, but they are not the same thing. The same sign can have different meanings, or the same meaning can be expressed by different signs. In mathematics (socioculturally constructed systems of signs and meanings) both conditions occur, particularly when we speak of quantity, digit and number.

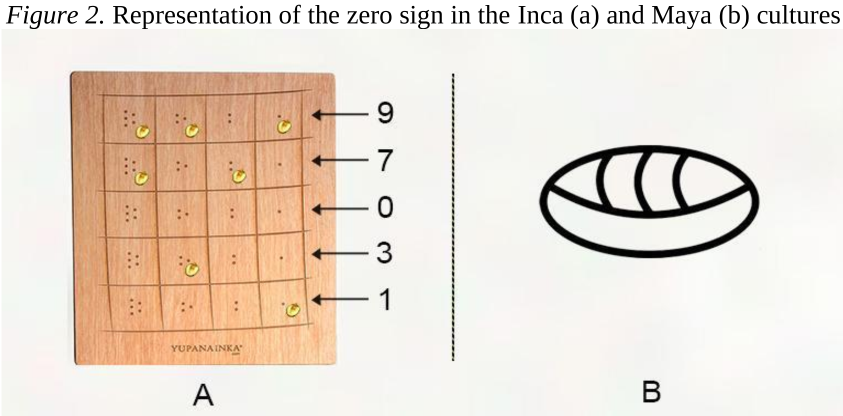
A sign whose meaning is quantities of something is not the same as one whose meaning is conceived as a digit, nor are both equivalent to one whose meaning is conceived as a number. A sign-quantity shows, evidences, points out one by one, a certain quantity of entities; a sign-digit refers to a set of entities whose quantity is expressed in the sign, it implies an order and a hierarchy with respect to other signs-digits; a sign-number, has its meaning defined by other signs and meanings of a specific mathematical system with rules, for example, the number zero. (Escotto-Córdova, 2021).

Figure 1. Representation of sign, quantity, digit, and number



In Figure 1 in the first cell, the sign-quantity shows a quantity of objects; in the second, sign-digit, the digit 3 means a set of a quantity of entities, and maintains an order and hierarchy with respect to other digits such as 2 or 4; in the third cell, sign-number, the digit 30 (thirty) carries a three followed by a zero, this sign “zero” in no way means an absent quantity, that is, “no physical entities.” It is not a sign of quantity, but a number sign whose meaning we paraphrase as follows: “sign that refers to the fact that the digit that accompanies it multiplies the quantity it represents the same times by the base of the numeral system,” in this case ten (3 x 10 = 30), and that gives a positional value of the digit in the tens, in the specific example.” That meaning is not given by the sign-digit, much less by the sign-quantity (Escotto-Córdova, 2021). The importance of this theoretical distinction will be clearly seen with the YITP and the representation of the zero on the board and the tokens with which the quantities of the decimal system are represented in the YITP. Its historical significance is that the Incas represented zero in the yupana by leaving an empty row (without tokens), and a space without knots in the khipu (Pereyra, 1990; Prem, 2016; Urton, 2005), this being a specific type of sign for zero (see Figure 2A) different from that used by the Maya in our continent (see Figure 2B).

Figure 2. Representation of the zero sign in the Inca (a) and Maya (b) cultures



The distinction between sign-quantity, sign-digit and sign-number is not an idle one; it implies fundamental conceptual and theoretical changes in the history of mathematics. The use of the sign as a quantity, as a digit or as a number empowers or restricts mathematical thinking, in Table 1 the subject of zero is analyzed.

Table 1. Example of table Meanings of sign-quantity, sign-digit and sign-number

Zero → sign-quantity	Zero → sign-digit	Zero → sign-number
0 (zero)		$3^0 = 1$
It means nothing, nothing, or entity, so that in 30, we understand 3 together with nothing results in three.	Zero means a positional place, so that in the digit 30, we never interpret three and nothing, but a set of thirty entities	Zero does not mean just a positional place, its meaning depends on a system with specific rules. In this case exponents indicate an operation in which its result is one

In general terms, we could say that in quantities, the elements that compose them are perceived one by one. That is why there is a biological limit common to several species of animals, in babies’ weeks old and in cultures and languages called “anumerical.” Its cortical foundation is usually located in the intraparietal sulcus (Chrisomalis, 2004; Dehaene et al., 2003; Dehaene et al., 2001; Everett, 2009; Lupyan & Bergen, 2016; Wassman & Dasen, 1994; Wiese, 2007).

If quantities are perceived, digits and numbers are conceived. The first step is to conceptualize sets of quantities, i.e., the notion of digits. “The Karitian people say, ‘take one’ and show another hand to name the digit six” (Everett, 2019, p. 80), thus used, the hand is a set of a certain number of fingers, i.e., a digit. The next historical step was to advance to the notion of “number,” that is, a sign whose meaning depends on other signs and meanings within a mathematical system with specific rules for its relationships. This step occurred with negative numbers and zero.

In the above examples we have cases in which the same sign has different meanings (as quantity, as digit and as number), which in itself complicates the understanding of arithmetic in children when they understand the digit-signs only as quantities of entities, and not as a set of quantities, or even worse, they confuse numbers with quantities.

Semiotic alternations have been present in every advance and development in the history of mathematics, in all times and in any culture. We can identify three main stages in the creation and use of signs to signify different mathematical concepts: the first, signs with the meaning of quantity: they represent by means of any figure or by means of objects each entity of a specific quantity, for example, to represent five put five points or five fingers. The second, signs with the meaning of digits: they use a figure or object to represent a set of entities, that is, the quantity as a set. For example, they use the hand as a set to represent five entities or fingers, or the human body to represent twenty. And the third, signs with meaning of numbers, are graphic signs or physical entities as engravings, whose meaning is defined by other sign-meanings of a mathematical system, for example, the vigesimal or the decimal, including the operations and relations between the sign-meanings. The clearest example is zero (Cajori, 2011; Escotto-Córdova, 2021; Dehaene, 2016; Ifrah, 2000; Menninger, 1992).

The signs at each stage of mathematical development used by different cultures and at different times have been varied: physical entities as a sign of quantities, for example each of the fingers of the hand, or parts of the body have been used from the representation of quantities with objects or things (Dehaene, 2016; Everett, 2019), or simply lines, drawn sticks. For example, the Ishango bone, from twenty thousand years ago, has marks of quantities representing one by one each counted entity (Evertt, 2019). In Egypt of the first millenia B.C., eight is represented by eight lines, sticks or vertical marks; the same in Iran, Elam culture, in the third millennium B.C.; the same in the Indian civilization of 2300-1750 B.C.; or the Hittite civilization, in Anatolia, between 1500-800 B.C.; in Greece between the 5th and 2nd century B.C.; and in the Lydian civilization between the 6th and 4th century B.C. (Ifrah, 2000; Menninger, 1992).

The transition to signs with meaning of a set of quantities is already noticeable with the Theban Greeks or the Chaldians who began to use a sign to represent the set of five between the 5th and the 1st century B.C.; the same was the case in the Lydia civilization between the 5th and the 4th centuries B.C.; also in Asia Minor, between the first half of the first millennium B.C.; and with Mayans between the 3rd and 4th centuries B.C. (Ifrah, 2000; Menninger, 1992).

Finally, an example of the transition from signs as a set of quantities to signs as numbers occurs with the notion of zero in India, in Central America with the Mayan culture representing zero with a drawing of an empty shell, or the cultures of Cambodia (Khmer culture), Viet-Nam, Laos with the representation of a point (Aczel, 2015).

3. Methods and materials

3.1. Tawa Pukllay Method in the Yupana (YITP)

Those who only focus on its instrumental function, overlook the semiotic nature (signs and meanings) of the YITP method and the yupana device itself, and consequently fail to recognize the power that signs and meanings carry as a “cognitive tool” (Lupyan & Bergen, 2016; Vygotski, 2017), particularly evident in numbers (Everett, 2019). In terms of the cognitive work involved in the use of YITP, it has the virtue of decreasing the working memory load with respect to that required in the Indo-Arabic arithmetic system, since YITP notation and operations depend on visuospatial relationships, pattern recognition, simple movements and a full-time visualization of quantities and performed operations. In theoretical-conceptual terms, it is useful for the understanding of the arithmetic system including zero as a number: YITP facilitates the distinction between sign-quantity, sign-digit, and sign-number.

According to Prem (2018a) the YITP is a board with squares arranged in 4 columns and 5 rows in which any number up to 5 digits can be represented: 0 to 99999. If it is required to work with larger quantities, rows can be added upwards representing the powers of 10^5, 10^6, 10^7, etc. In each row the cells are marked from left to right with 5, 3, 2 and 1 dots. The numbers are represented by placing small objects such as seeds (tokens) in the cells. Each row in the board represents a digit. It starts with the bottom row to represent the digit for units; the next row up for the tens digit and so on, up to the ten thousand. According to this rule, the digits 1, 2, 3 and 5 are represented with a single token with quantity-sign function (one entity), which is placed in the cell with the dots engraved on it. This is to represent the sign-digit of one, two, three and five entities. The cell is the sign conformed by a set of dots whose rule indicates that it can be used in writing a number by placing only one token on it or none at all. For example, the digit 7 is represented with one token in cell 5 and one token in cell 2. Each row of the board represents a positional value from bottom to top with the meaning of units, tens, hundreds, etc., for example, the number 47, with a token in cells 5 and 2 in the bottom row and the next row up with a token in cells 3 and 1 (see Figure 2A).

With these elements we can already distinguish signs-quantities, signs-digits, and signs-numbers. The tokens from right to left and from top to bottom represent the order and hierarchy of the digits, and that their meaning is numerical, since they depend on the YITP system of digit-signs with exponential multiples with base 10. Now, since in each cell one or no token is placed to indicate quantities (the token is not counted, but indicates the number of points indicated in that cell), they only signify quantities, which located in one or more cells of a row form a group of digit cells of the YITP. That is, each row represents a digit that in turn, depending on its order and the rules of the numeral system (base 10 and exponential multiples), will form part of a number-sign.

Zero, in its double condition of nothing and number is clearly evident when we have a multi-digit number, for example 97031 (see Figure 2A), which contains representations of digits that are arranged in five rows of the YITP, having in the top row the representation of nine, then the representation of seven, no token in the third row, and in the rows below are represented three and one respectively. All of the above decreases the working memory load and facilitates the theoretical-conceptual understanding of the notions of "number," "digit," "quantity" and the importance of zero in a certain place value.

The semiotic properties of the YITP, both numerical and didactic, can be a valuable tool for the reading of numbers in elementary school children both in the city and in rural places, but above all, in those who due to their socioeconomic, cultural and social conditions have not had more academic support than that provided in their schools by their teachers and friends. Therefore, we proposed to provide evidence of the sociocultural usefulness for rural education in a context of pandemic, which improves the speed and effectiveness in reading the inca numeral system in the YITP and its equivalence to the Indo-Arabic system through a self-learning serious game on an electronic tablet used by the children of the community.

3.2. Serious Game SER0-TP

A serious game is an educational application whose main purpose is to coherently combine serious aspects such as teaching, learning, communication or even information with fun aspects of video games in a non-exclusive, non-exhaustive way (Alvarez, 2007).

Education researchers have taken a keen interest in gamification since 2013 (Dominguez et al., 2013). The gamification in education is an intense and quickly developing area of research, with hundreds of new relevant publications coming out every year (Lee & Hammer, 2011). Gamification has also been shown to have favorable results, relating its use to greater student engagement and learning (Tsai et al., 2019; Díez et al., 2017)

Studies show that gamification can make a positive contribution to the education process (Kim & Castelli, 2021; Manzano-Leon et al., 2021; Swacha, 2021). Serious Game is a wide field that may be used for many educational purposes. Since it is also a mean of entertainment, multiple learning objectives can be covered while many skills are developed at same time: information technology, communication, language and actually almost any field. And one very important thing, specially committed to fulfill the multigrade schools' needs: It is for all ages. (Mouaheb et al., 2012). Learners with different skills can participate effectively in the same learning application (Sezgin et al., 2018) and the more they get engaged, the more they understand their own learning process. This process stimulates student's autonomy at learning time in a more effective way (Lee & Hammer, 2011).

The SER0-TP serious game installed on an electronic tablet was designed following the learning guidelines proposed by the sociocultural theory of learning and development (Vygotski, 1995; Vygotski, 2010; Vygotski, 2017), Galperin's theses with his theory of knowledge formation by stages (Galperin, 2009a; Galperin, 2009b; Galperin, 2009c; Talizina, 2009), and Leontiev's activity theory (Leontiev, 1984; Leontiev, 2009). These guidelines consist in the fact that psychological development and learning are always carried out with the help of others, by others, for others until the moment arrives when the individual (child or adult) does it for himself as if he were someone else using his internal language. Its practical expression is self-learning. Therefore, learning takes place in stages, some external (social, object, figurative-drawings, etc.) and other internal (oral language: one speaks when learning; and another silent: one speaks to oneself silently). In the research, the external aspect was the initial orientation of the teacher and the tutorials, the object aspect was the electronic tablets, and the internal and playful aspect was the game played by each child at his own pace and taste as a manifestation of his self-learning.

The design of the mechanics, dynamics and aesthetics, the guidelines proposed for MDA (Hunicke et al., 2004) were used. Figure 3 shows Module 1: PUKLLAY, which contains the interactive exercises, as well as the children in the middle of learning.

The tablet was programmed to record frequencies of use for each level, successes, times, etc., whose numerical data were later analyzed and statistically processed.

3.3. Participants

The research was carried out in the community of Huamachuco, where there are approximately 25 families (25 fathers, 25 mothers and an average of between two and three children per family), for a total of approximately 125 inhabitants, most of whom work in rural agriculture and are illiterate. The language spoken by the majority is Quechua (Cañaris variant). The families live in single room houses, separated in a distance around 10 to 20 minutes by walking from each other (there is no car or bus transportation within the community, except for a few motorcycles that are used for very specific purposes). The twenty children have their fathers and mothers at home. Twelve children (4 boys and 8 girls) from the first four grades of primary school enrolled in the educational institution Nº 10244 multigrade of the community of Huamachuco (Peru), participated in the project. The learning process lasted approximately two months.

The selection of the sample was intentional and exhaustive: all the children of the community who were present and agreed to participate were divided into two groups: those children of 1st and 2nd grades, who never had had any classroom learning experience as a result of the quarantine due to the COVID-19 pandemic and children of 3rd and 4th grades who already had knowledge of the Indo-Arabic numeral system because of their past two years studying at regular school, see Table 2.

Table 2. Boys and girls by academic grade and age

Grade	Boys (age)	Girls (age)	Total
1º		1A,1B (6 years)	2
2º		2A,2B,2C (7 years)	3
3º	3B,3C (8 years)	3A (8 years)	3
4º	4B, 4D (9 years)	4A, 4C (9 years)	4

3.4. Experimental design

The experimental design, as shown in Table 3, worked with 3 types of variables: (1) the independent manipulated variable (IMV), which is a variable that can be manipulated in its magnitudes, frequency, the sequence of its presentation, the time in which it is presented and how long it lasts, it can be placed and removed at the experimenter's will in each subject or group, etc. For the research we considered the YITP programmed on an electronic tablet under the criteria of a serious game which we have called SER0-TP. (2) the independent variable of selection (IVS), which is any variable whose only possible manipulation is to select that it is present or absent in a group, we have considered the Self-learning Group without previous presential learning experience (1st - 2nd grade) and the Self-learning Group with previous presential learning experience (3rd - 4th grade). And finally the following were considered as dependent variables to measure the learning process: (3) Digit reading speed rate (DRSR): digit reading speed measured in seconds per digit, Zero read attempt ratio (ZRAR).

The empirical work of the Tawa Pukllay method has been disseminated by Asociación Yupanki (Dhavit Prem and Alvaro Saldívar) through workshops aimed at students and teachers at both city and community levels in Peru and has been presented in several international events in Colombia (Saldívar et al., 2019a), México (Saldívar et al., 2019b) and Peru (Saldívar, 2019). Also some other events are National Council of Science, Technology and Technological Innovation of Perú (CONCYTEC) 2015; National Library of Perú (BPN) 2017, National Institute of Peruvian Culture INC-Cusco, 2016; Science & Engineering Festival in Washington DC

2017; Latinoamerican Congress of Mathematics (RELME) 2017 and 2018; VI International Congress of Ethnomathematics 2018; High School Academy Congress Guatemala 2017.

Table 3. Learning parameters

Group	Independent variable of selection (IVS))	Independent manipulation variable (IMV)	Dependent variables
Group 1	Self-learning Group without presential learning experience (1^{st} - 2^{nd} grade)	SER0-TP	FAAR DRSR ZRAR
Group 2	Self-learning Group with past presential learning experience (3^{rd} - 4^{th} grade)	SER0-TP	FAAR DRSR ZRAR

Our research was planned to provide theoretical and empirical systematization, as well as support from educational institutions in Peru (Universidad de Lima) with the participation of researchers from Mexico (Facultad de Estudios Superiores Zaragoza, Universidad Nacional Autónoma de México), for the rescue of the YUPANA INCA as a didactic resource for the teaching of mathematics at the elementary level. The research involves the learning of technology inaccessible to children in the community. On the other hand, it has been proposed to continue the support to this community, but in a face-to-face way, when the restrictions of the pandemic or COVID-19 are lifted.

The community was visited, the research proposal was presented and a discussion was held with the adults seeking approval for the participation of their children in the research in a community session with the entire community. The facilitator was the multigrade teacher of Educational Institution 10244. Once accepted, their informed consent about the research was confirmed by the inhabitants of the community of Huamachuco of the Cañaris community, the parents gave their informed consent forms signed and received a SER0-TP kit (electronic tablet, serious game and solar charger), YITP kits (book, physical yupana and tokens) and instructions for use. Biosafety protocols were observed during the pandemic.

Before starting the experimentation, in order to discard cognitive difficulties, each child was psychologically evaluated by the specialist, Dr. Alejandro Escotto-Córdova through the review of previous videos of the interviews to the children and the application of a brief cognitive assessment designed for the andean population. This assessment had as reference the Montreal Cognitive Assessment - MoCa test. Also, our research team designed a test called “*Yupay Tupay - Sami*” of attitude-emotion towards mathematics based on the Attitudes Toward Mathematics Inventory Test (ATMI) and the representation of responses on a Likert scale with five emotions (very sad, sad, indifferent, happy and very happy) (see Appendix 1) which was applied in order to know the initial and final perception (attitude) of all students towards mathematics. Finally, all children received at least two teacher-guided sessions on how to use the tablet and an introduction on how to play with the SER0-TP game.

No child presented cognitive dysfunction, despite the fact that some were below the norm. This is explained by specialist Dr. Alejandro Escotto-Cordova because being outside the statistical norm, being statistically abnormal, does not necessarily imply being disordered, dysfunctional, sick, suffering from some pathology or developmental incompatibility. It is simply not being like the others to whom the individual is compared. Certainly, any disorder, cognitive dysfunction, pathology or developmental incompatibility implies being outside the norm, but not the other way around. The only data provided by applied statistically standarized tests,is how close or far the individual is from the statistical norm derived from the population sample. Nothing more. Measuring is not diagnosing (Escotto-Córdova et al., 2021). The diagnosis of cognitive dysfunction is suspected when the individual who is below the statistical norm in a test, also presents notable difficulties in learning with the help of others and with new didactic strategies. This was not the case in any of the Cañaris children, and they even learned to read Indo-Arabic numbers only by playing with the electronic tablet and the YITP.

Once all the above was finished, the children were not visited again in their community until after two months in which the data from the tablets were collected, so the children played freely with the tablet as long and as often as they wanted. Remote monitoring was available based on simple phone calls to their teacher for any eventual technical problem support. Only one error occurred due to forgetting the password to access the electronic tablet but it was solved remotely. The accuracy of answers within the game, the scores, time and frequency of use of the electronic tablet was automatically recorded by SER0-TP without the children being aware of it.

Figure 3. SER0-TP Pukllay module and children learning

The experiment consisted of:

- The children had to watch some videos included inside the SER0-TP, where lessons on how to read and write numbers on the yupana board were detailed explained.
- After watching the videos, the children could enter the game, where numbers represented in the yupana board using the YITP method (inca numerical system), should be read, interpreted and written using the Indo-Arabic numerical system by console (see Figure 3). A total of 60 exercises had to be solved: 12 exercises with 1 digit, 12 with 2 digits, and so on up to 12 exercises with 5 digits.
- After solving the 60 exercises, a *congratulations* message would appear saying that the task is over.
- Majority of children found out that by reentering the game and by canceling the end message, it was possible to continue playing it, so they decided to do it, performing many unexpected additional exercises, which also were recorded by the system and now are part of the analysis under the label *UAE*.

External factors to consider:

- Children could not be helped by their parents since most of them are illiterate, much less they knew how to deal with any electronic device.
- The teacher taught them the basic principles for taking care of the tablets (turn on, turn off, entering the SER0-TP, watching videos, running the game and loading batteries). No further lessons were imparted during the whole experimental process.
- In order to make the application SER0-TP more attractive, familiar and understandable to children, the whole design considered pictures of the children themselves, text and voice feedback messages in local quechua language and characters such as avatars, dresses and other signs based in ancient local cosmovision.
- One of the most important points considered at the moment of designing the current experiment, was the reincorporation of the YITP method, which is a recent proposal of rediscovery of the inca's math after 500 years, which uses the inca board for calculations, the inca numerical system, pattern recognition, andean principles and quechua names for token movements (Prem, 2018a). It means a totally different way of reading numbers and performing arithmetical operations than Indo-Arabic classic method currently used worldwide.
- After the experimental process, the teacher extracted the files containing the hidden records (XML files) of exercises performances, times and scores and sent them back to the central in Lima for analysis.
- The teacher also interviewed the children who said that those who had siblings at home or friends living near, could help each other on learning how to use the buttons, enter the videos and the game. They also said that each child did his own exercises because they wanted to. No pressure of time existed, nor regular basis of practice was imposed. There was only a general and open suggestion of "it would be good if practice would occur half an hour a day until you finish the exercises." This suggestion was exceeded because all the children said they enjoyed learning in game mode.

4. Results

At the end of the research, the data recorded from the interaction between the children and SER0-TP was collected for the analysis of the following metrics which include two scenarios: expected exercises scenario (EE) and additional unexpected exercises scenario (UAE), which is a series of exercises that children decided by themselves to solve even after finishing the expected tasks (more exercises than requested).

- First attempt accuracy ratio (FAAR): percentage of number reading accuracy at the first attempt.

- Digit reading speed rate (DRSR): digit reading speed measured in seconds per digit (includes the seconds of reading a digit written in the Inca numeral system and its equivalent writing in the Indo-Arabic system using the SER0-TP). This rate is important because it allows us to observe the learning process of the children and the internalization of the value of the squares that gradually replaced the continuous counting of dots.
- Zero read attempt ratio (ZRAR): percentage of accuracy in reading numbers containing at least one zero as a digit at the first attempt.

The FAAR values curves of the reading numbers exercises containing between 1 to 5 digits show that some of them are ascending and others sinuous during the learning process, in which almost all the children achieved above the 90% FAAR, except for the three 3rd grade boys and one 2nd grade girl. However, these four children also showed upward learning, particularly boy 3B who starts with 8% FAAR and ends with 75% FAAR. Child 3A starts with 75% FAAR at 1-digit reading, and then she gets a 50% FAAR at 2 or more digits reading, apparently due to difficulties when identifying the positional value of the represented digits.

From the results obtained it is worth noting that 1st and 2nd grade children learned the Inca numeral system by reading in an autonomous and playful way and its equivalence to the Indo-Arabic system through the SER0-TP in a range of time <20’11” – 2h 26’ 28”>, also showing a FAAR increase above the 90% for children who also solved the unexpected additional exercises UAE. The range of SER0-TP usage time of all children is <13’44” - 5h 41’37”> (see Table 4).

It is worth noting that although child 2C started with 0% FAAR at 1-digit reading, he increased his FAAR noticeably, completing all the EE and even performing unexpected additional exercises UAE, where he reached a 91% FAAR. The total SER0-TP usage for this child was around 2.5 hours (see Table 3). It is important to highlight that children 2C, 3B, 4B and 4D, started with the lowest FAAR (0%, 8%, 17% and 25%) and ended with FAAR: 91%, 75%, 100% and 90%, respectively.

Table 4. FAAR per digit and total times of SER0-TP usage (EE & UAE)

	Expected Exercises (EE)							Unexpected Additional Exercises (UAE)		
Student	Qty	Time EE	1-digit FAAR	2digits FAAR	3digits FAAR	4digits FAAR	5digits FAAR	Qty	Time UAE	FAAR
1st Grade										
1A (♀)	49	33’39”	58%	83%	92%	83%	100%	0	---	---
1B (♀)	60	19’17”	75%	83%	83%	92%	75%	294	1h 43’20”	92%
2nd Grade										
2A (♀)	60	11’25”	100%	83%	100%	100%	92%	25	8’46”	92%
2B (♀)	60	34’43”	75%	100%	92%	75%	83%	0	---	---
2C (♀)	60	31’50”	0%	42%	83%	83%	58%	319	1h 54’38”	91%
3rd Grade										
3A (♀)	60	54’04”	75%	50%	58%	42%	67%	203	1h 55’27”	74%
3B (♂)	40	13’44”	8%	58%	83%	75%	75%	0	---	---
3C (♂)	60	22’03”	67%	67%	92%	75%	75%	8	4’08”	71%
4th Grade										
4A (♀)	60	28’02”	92%	92%	83%	92%	67%	813	5h 13’35”	90%
4B (♂)	60	21’35”	17%	92%	83%	100%	92%	4	1’19”	100%
4C (♀)	57	25’55”	100%	100%	58%	75%	100%	0	---	---
4D (♂)	60	24’31”	25%	100%	92%	75%	100%	407	2h 52’45”	90%

Note. The times shown do not consider the minutes spent watching the video tutorials; only the effective time of the exercises they performed was counted.

4.1. Digit reading speed rate (DRSR)

At the suggestion of the authors of the YITP method and methodology, with 8 years of teaching experience, three time ranges were considered: < 0-5]s, < 5-10]s and < 10+>s. The first range corresponds to the DRSR of subitizing (Cheeseman et al., 2021), i.e., children do not need to count and only seeing the marked cells they recognize the represented digit, so it is very fast and constitutes an excellent resource of semiotic alternation to introduce Indo-Arabic arithmetic; the second range corresponds to the counting DRSR (children may already be subitizing in some cases, but they still need to count dots, so reading is slower); the third range corresponds to the counting DRSR with difficulties and/or typing errors when entering the result, which imply an increase in time because the number needs to be entered again in the answer (see Figure 4 and 5).

Figure 4. Digit reading speed rate (DRSR) for 1st and 2nd grade - Expected exercises

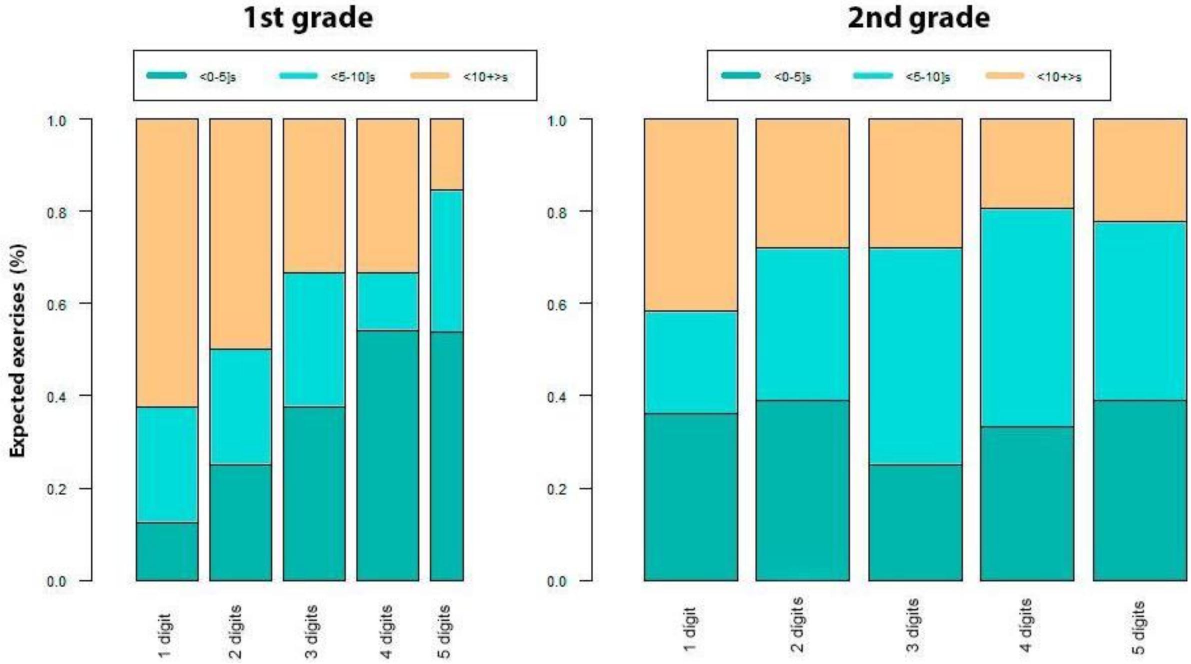
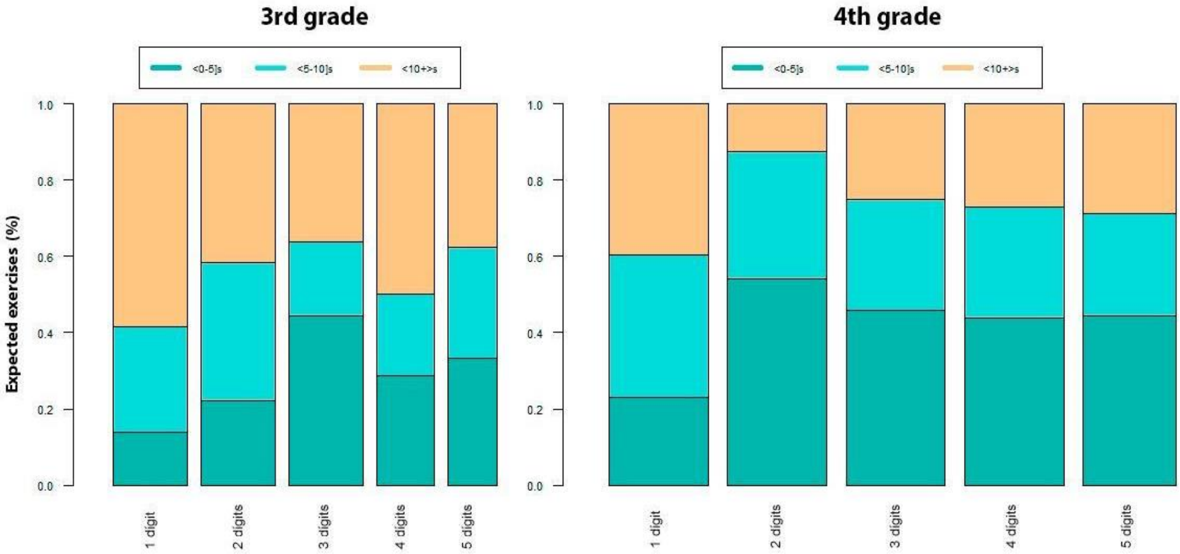


Figure 5. Digit reading speed for 3rd and 4th grades (Expected Exercises)



Note. Range < 0-5]s in dark green, < 5-10]s in light green, and range of < 10+>s in light orange. The thickness of the bar indicates proportionally the number of exercises performed per number of digits.

In all grades it is observed that there is a growth in the exercises solved in the range < 0-5]s, while in contrast, a decrease is seen in the range < 10+>.

Comparing Figure 6 of UAE with the previous EE graphs (Figures 4 and 5), it is observed that all grades increase the DRSR in the < 0-5] interval, exceeding 70%, with the exception of 3rd grade which rises from 33.33% to 52.15% of DRSR in the < 0-5]s interval.

Figure 6. Digit reading speed rate - Additional exercises

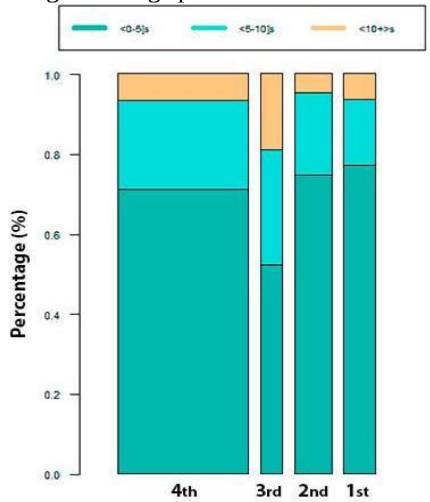
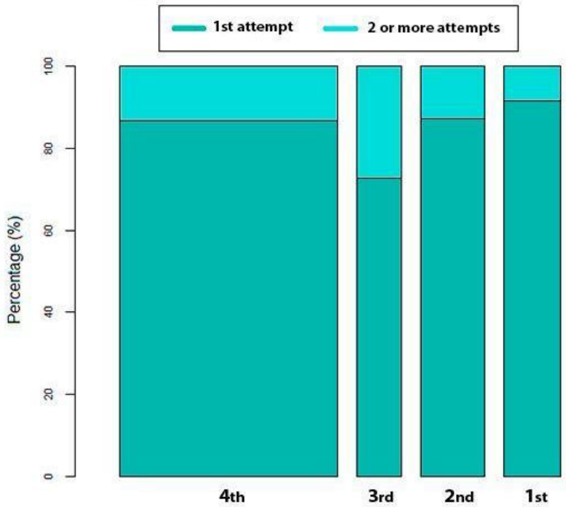


Figure 7. Reading accuracy of numbers containing at least one zero as a digit



Note. Numbers that were read correctly at the first attempt are shown in dark green, while those that required two or more attempts are shown in light green. The thickness of the bars indicates respectively and proportionally the total number of these exercises, which vary by grade because they were generated randomly.

4.2. Percentage in reading numbers containing at least one zero as a digit at the first attempt (ZRAR)

To determine the reading accuracy of numbers containing at least one zero as a digit at the first attempt, the expected EE and additional UAE exercises were included, see Figure 7.

It is observed that the percentage of numbers containing at least one zero as a digit read correctly at the first attempt exceeds 85% for all grades, with the exception of 3rd grade, which reaches 72.73%. The grades with the highest percentages are 1st and 2nd grade with 91.67% and 87.32% respectively.

5. Discussion

Our research suggests that children in a remote community, with educational and technological backwardness, in conditions of temporary isolation due to the pandemic, without a teacher, and with only two months of self-generated play activity, have no difficulty learning two new contents: (1) to use modern technology and, (2) to learn the logic of Inca mathematics installed in a tablet. YITP was shown to be a powerful semiotic alternation that embedded in an electronic tablet (SER0-TP) fosters effective and fast learning, based on a scheme that stimulates playful self-learning.

There is no significative difference when comparing 1st and 2nd grades versus 3rd and 4th grades FAAR. This would suggest that it is not a requirement to have previous basic arithmetical knowledge in order to learn the YITP using the SER0-TP, actually it can be also observed that children from 1st and 2nd grades have achieved, at the end, better FAAR than children from 3rd grade. The similar DRSR of 1st and 2nd versus 3rd and 4th grades and the higher DRSR growth which corresponds to 1st grade, would indicate that previous knowledge of Indo-Arabic system of numbers could demand more time when learning YITP because of cognitive reorganization. ZRAR is high for all four grades and there is no significative difference when comparing the 1st and 2nd grades versus 3rd and 4th grades. This would suggest that YITP would be useful when reading numbers which contain at least one zero.

We argue that in the Quechua culture, in Peru, zero was also represented in the Inca Yupana, but as an absence of quantity in a visuospatial matrix (Prem, 2018b). This representation of zero is intuitive by perceiving the absence of quantity together with its positional value in space, which facilitates performing operations on quantities and decimal-based numbers. Our research suggests that these intuitive properties of the YUPANA INCA for understanding zero are shown in the rapid identification of Indo-Arabic numbers with zeros from using the YUPANA INCA on the electronic tablet, for example 85% of reading numbers containing at least one zero as a digit were read correctly on the first attempt for all grades.

The reading of numbers containing at least one zero as a digit deserves special analysis, since in the Indo-Arabic system it represents a particular challenge for students to distinguish zero with quantitative significance due to its positional value. Nevertheless, in the present investigation a high percentage of reading accuracy was observed on the first attempt of numbers containing at least one digit zero (ZRAR).

The YUPANA INCA used as a didactic resource meets the requirements of the historical-cultural model of the learning process developed by Vygotski et al. (Galperin, 2009a; Galperin, 2009b; Galperin, 2009c; Vygotski, 1995; Vygotski, 2010; Vygotski, 2017; Leontiev, 1984; Leontiev, 2009; Talizina, 2009).based on the transition from object representation to symbolic representation to the internalization and mastery of knowledge, and a few brief explanations and test exercises are sufficient for it to be used playfully as self-learning. Our results provide evidence that this process facilitated by the YUPANA INCA, from:

- Translating quantities into expressions: SER0-TP consists precisely in counting points in the Inca numeral system to be then represented by Indo-Arabic numbers. For this purpose, SER0-TP makes use of the matrix structure of the YITP (see Figure 2), which allows counting dots, adding the quantities of dots from other cells and even accelerating the perceptual process of suddenly recognizing quantities of no more than five objects.
- Using calculation strategies and procedures: SER0-TP allows to easily develop divergent thinking and the associative arithmetic property from the very learning of counting, for example when 5 and 3 dots cells are activated together to represent the number eight.
- Create numerical relationships: SER0-TP stimulates children in the recognition and differentiation of quantities represented in multiple rows, their corresponding positional values and their equivalence in the Indo-Arabic system and facilitates the understanding of zero by showing the sign “empty row” when a power of ten is represented.
- Identifying movements and locating patterns: SER0-TP uses the logic of YITP, which is precisely a proposal for solving arithmetic operations based on the execution of token pattern recognition and the use of a set of moves that depend on the location of these tokens. It is important to highlight that SER0-TP has allowed children in the 1st and 2nd grades of primary school, who in the context of the pandemic did not have the presence of a teacher or tutor for a year and a half, to learn autonomously the numeration of up to five digits. SER0-TP appeals to the autonomy of each child to decide how far to advance, regardless of the grade level, unlike the proposal of current curricula that encourage collective learning.

Our findings show that children have achieved in some cases better levels of digit reading speed at the first attempt RSDR and the percentage of digit reading accuracy at the first attempt FAAR than other children of higher grade, which would demonstrate that with this methodology it is not necessary to limit the learning of number reading to a certain number of digits, and that on the contrary, YITP improves children’s reading accuracy and speed the more digits they have and the greater the number of exercises they solve, as opposed to what is suggested by the official curriculum that promotes learning by segmenting the number of digits that should be learned according to each school grade: 1st grade up to the number 20, 2nd grade up to the number 20, 3rd grade up to three digits, 4th grade up to four digits and 5th and 6th grades up to six digits.

The development of autonomy in students has been highlighted as a goal of mathematics education (Ben-Zvi & Sfard, 2007; Yackel & Cobb, 1996). Learner-centered teaching strategies such as mathematics instruction based on real-life contexts, inquiry-based learning, and problem-centered learning (Cobb & Yackel, 1996; Wheatley, 1994) have been discussed to increase learner autonomy in mathematics learning. Intellectual autonomy has been defined as, "students' awareness and willingness to draw on their own intellectual capacities when making mathematical decisions and judgments" (Cobb & Yackel, 1998, p. 170). Given the pandemic situation, this ceased to be a proposal and became a necessity, a necessity that due to the results obtained shows to be feasible and at the same time a new opportunity that promotes active self-learning.

The results of the adapted attitude test showed an individual improvement, so it is presumed that these mood conditions together with the children's enthusiasm and curiosity favored self-motivation, which added to the acceptance of the SER0-TP semiotic ludic instrument, allowed the children to achieve the results obtained in a short time.

6. Conclusions

The parameters used in the measurement (FAAR, DRSR and ZRAR) indicate that children in 1st and 2nd grade using the YITP as a didactic instrument learned to read Indo-Arabic numbers. We recommend that the YITP be used as a didactic support in the teaching of arithmetic in the early years in both rural and urban schools. Children in the early grades, even if they have no experience with the Indo-Arabic system, can learn to read using the Yupana YITP as a semiotic alternation. Young children who have not previously had access to technology and who live in isolated conditions can learn to handle technology with playful software in a short time.

The present investigation considers only children from 1st to 4th grade of elementary school; it is suggested to carry out investigations in urban schools and in other rural areas to contrast the results obtained in the present investigation. Although we had a teacher who facilitated the delivery of the SER0-TP kit, and helped in the realization of the cognitive evaluations and initial tests, it would be ideal for specialists in the psycho-pedagogical area as well as YITP to have direct access to children in future investigations, a fact that could not be done on this occasion due to the limitations of the pandemic.

Rural single-teacher requires didactic strategies and tools to support the individual needs of each student to learn number sense and the notion of place value. It is essential to develop these initial skills that will mark the long-term educational trajectories of these children in mathematics.

Although the present study is only focused on quantity recognition, learning to read numbers in the Inca numeral system and its equivalence to the Indo-Arabic system, it constitutes the basis for a second serious game included in the SER0-TP that will be discussed in a following article.

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Appendix 1

Test “Yupay Tupay – Sami”

1.- Las matemáticas me hacen sentir... [Mathematics makes me feel…]					
2.- Cuando doy un examen de matemática, me siento... [When I have a math test, I feel…]					
3.- Cuando veo que las cosas que quiero hacer necesitan matemáticas, me siento… [When I see things that I want to do need math, I feel…]					
4.- Conociendo nuevos temas de matemáticas me siento… [Learning new math themes I feel…]					
5.- Cuando estoy estudiando matemáticas me siento… [When I study maths, I feel…]					
6.- Cuando me dan un problema de matemáticas me siento… [When they give me a math problem, I feel…]					
7.- Cuando pienso que las matemáticas tienen muchos más temas por descubrir, me siento... [When I think that maths have many more themes to discover, I feel…]					
8.- Cuando pienso que de grande trabajaré usando las matemáticas, siento... [When I wonder that being older I will work with maths, I feel…]					
9.- Cuando tengo que resolver un problema de matemática solo, me siento… [When I have to solve a math problem alone, I feel…]					
10. Cuando se acerca un examen de matemática, me siento… [When a math test is coming up, I feel…]					

Disfrute [Enjoy]	Motivación [Motivation]	Autoconfianza [Self-confidence]	Valor [Value]	Ansiedad [Anxiety]